Problem 1.1 ECEN 3613

$v = 330\ m/s$, $f = 2\ kHz \rightarrow \omega = 2\pi f = 4\pi \times 10^{3}$

$\phi_0 = 36°$

$P(x,t)\Big|_{\substack{x=0 \\ t=50\mu s}} = 10\ N/m$

$$P(x,t) = P_{max} \cos\left[\omega\left(t - \frac{x}{v}\right) + \phi_0\right]$$

$$= P_{max} \cos\left[4\pi \times 10^{3} t - \frac{4\pi \times 10^{3}}{330} x + \underbrace{36°}_{0.2\pi}\right]$$

$$p(0, 50\times 10^{-6}) = 10 = P_{max} \cos(0.2\pi + 0.2\pi)$$

$$\Rightarrow P_{max} = \frac{10}{\cos(72°)} = 32.36$$

$$\rightarrow \quad p(x,t) = 32.36 \cos\left[4\pi \times 10^{3}\left(t - \frac{x}{330}\right) + 36°\right] \ N/m^2$$

Problem 1.5

$$y(x,t) = 1.5 \sin(0.5t - 0.6x)$$

$$\rightarrow \omega = 0.5 \quad , \quad v_p = \frac{5}{6} \ m/sec$$

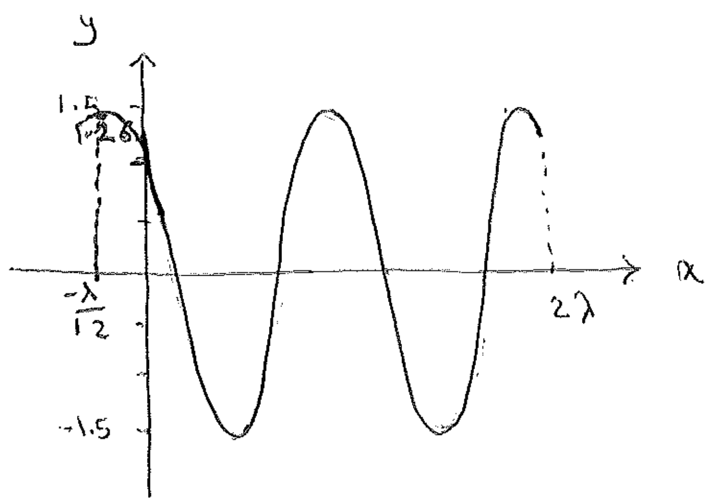
$$f = \frac{\omega}{2\pi} = \frac{1}{4\pi} \quad , \quad \lambda = \frac{v_p}{f} = \frac{5/6}{\frac{1}{4\pi}} = \frac{20\pi}{6} = 10.47 \ m$$

$$y(x,t)\Big|_{t=2} = y(x,2) = 1.5 \sin\left(1 - \frac{3}{5}x\right)$$

$$\rightarrow y(x,t)\Big|_{\substack{x=0 \\ t=2}} = 1.5 \sin\left(1^{(rad)}\right)$$

$$= 1.5 \sin(57.3^\circ)$$

$$= 1.26$$



$y_{max} = 1.5$

at $x = ?$

$$x = ? \Rightarrow 1 - \frac{2\pi x}{\lambda} = \frac{\pi}{2}$$

$$x = \frac{-\lambda}{12}$$

Problem 1.11

$y_1(t) = 3\cos\omega t$
$y_2(t) = 3\sin(\omega t + 60^\circ)$

$\sin(x + 90^\circ) = \cos x$, $\sin x = \cos(x - 90)$

$\rightarrow \sin(\omega t + 60) = \cos(\omega t - 90 + 60) = \cos(\omega t - 30)$

$\rightarrow y_2(t) = 3\cos(\omega t - 30^\circ)$

$\rightarrow y_2(t)$ lags $y_1(t)$ by 30°

Problem 1.13

at depth x_1, the amplitude is $A_1 = k e^{-\alpha x_1}$

at depth x_2, the amplitude is $A_2 = k e^{-\alpha x_2}$

Taking the ratio $\rightarrow$ $\frac{A_1}{A_2} = \frac{k e^{-\alpha x_1}}{k e^{-\alpha x_2}} = e^{\alpha(x_2 - x_1)}$

$(\ln e^x = x)$ $\rightarrow \alpha(x_2 - x_1) = \ln \frac{A}{A_2}$

$\rightarrow \alpha = \left(\frac{1}{x_2 - x_1}\right) \ln\left(\frac{A_1}{A_2}\right)$

$= \frac{1}{90} \ln\left(\frac{98.02}{81.87}\right)$

$= 2 \times 10^{-3}$ $\frac{\text{nepers}}{\text{meter}}$

Problem 1.15

$z_1 = 3 - 2j$

$z_2 = -4 + 3j$

a) $|z_1|^2 = 9+4 \rightarrow |z_1| = \sqrt{13}$, $\angle z_1 = tg^{-1}\left(\frac{-2}{3}\right) = -tg^{-1}\left(\frac{2}{3}\right) = -33.7^\circ$

$|z_2|^2 = 16+9 \rightarrow |z_2| = 5$, $\angle z_2 = tg^{-1}\left(\frac{3}{-4}\right) = 180 - 36.87 =$

143.1°

b) $|z_1|^2 = z_1 z_1^* = (3-2j)(3+2j) = 9+4 = 13$

$\rightarrow |z_1| = \sqrt{13}$ (sam)

c) $z_1 z_2 = |z_1||z_2| \underline{/ (\angle z_1 + \angle z_2)} = (\sqrt{13})(5) \underline{/ (-33.7+143.1)} = 18.03$

$\times \underline{/ 109.4}$

d) $\dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|} \underline{/ (\angle z_1 - \angle z_2)} = \dfrac{\sqrt{13}}{5} \underline{/ ((-33.7-143.1)} = 0.721 \underline{/ -176.8}$

e) $z_1^3 = |z_1|^3 \underline{/ 3\angle z_1} = 46.87 \underline{/ -101.1^\circ}$

Problem
1.18

$z_1 = 5\angle -60^\circ \qquad z_2 = 2\angle 45^\circ$

a) $z_1 z_2 = (5 \times 2)\ \angle(-60+45) = 10\angle -15^\circ$

b) $z_1 z_2^* = (5 \times 2)\ \angle(-60-45) = 10\angle -105^\circ$

c) $\dfrac{z_1}{z_2} = \dfrac{5}{2}\ \angle(-60-45) = 2.5\angle -105^\circ$

d) $\dfrac{z_1^*}{z_2^*} = \dfrac{5}{2}\ \angle(60-(-45)) = 2.5\angle 105^\circ$

e) $\sqrt{z_1} = \sqrt{5}\ \angle -\frac{60}{2} = \pm 2.24\angle -30^\circ$

$= \left[2.24\angle -30^\circ ,\ 2.24\angle +150\right]$

Problem
1.20

$z = 3-4j$

$e^{z} = e^{3-4j} = e^{3} e^{-4j} = e^{3}\angle -4(\text{rad})$

$= 20.09\angle -229.2^\circ = 20.09\angle 130.8^\circ$

other way: $e^{3} e^{-4j} = e^{3}\left[\cos(-4) + j\sin(-4)\right]$

$= 20.09(-0.654 + j\,0.757)$

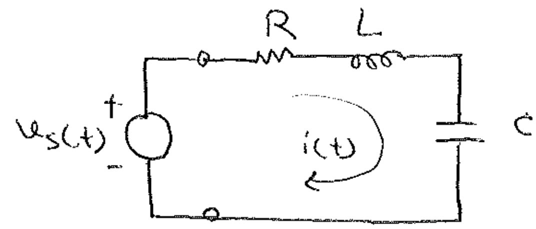
$= -13.13 + 15.20j$

$= 20.09\angle \mathrm{tg}^{-1}\left(\frac{15.20}{-13.13}\right) = 20.09\angle 180-\mathrm{tg}^{-1}\left(\frac{15.20}{13.13}\right)$

$= 20.09\angle 130.8$

Same result

Problem 1.24



$$v_s(t) = v_0 \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$\tilde{v}_s = v_0 \angle \frac{\pi}{3} = v_0 e^{\frac{\pi}{3} j} = v_0 e^{60^\circ j}$$

a) $$v_s(t) = R\, i(t) + L \frac{di(t)}{dt} + \frac{1}{C}\int_{0^-}^{t} i(t)\,dt + v_c(0^-)$$

assum:
$v_c(0^-) = 0$

$\rightarrow$ b) $$\tilde{V}_s = \left(R + \omega L j + \frac{1}{\omega C j}\right)\tilde{I}$$

c) $$\tilde{I} = \frac{\omega C j}{(1-\omega^2 LC) + \omega RC j}\tilde{V}_s = \frac{\omega C V_0}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}} \angle 60^\circ + 90^\circ - tg^{-1}\left(\frac{\omega RC}{1-\omega^2 LC}\right)$$

or:

$$\tilde{I} = \frac{1}{R + \left(\omega L - \frac{1}{\omega C}\right) j}\tilde{V}_s = \frac{v_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \angle 60^\circ - tg^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Problem 3.1

starting point $(1, -1, -3)$
ending point $(2, -1, 0)$

vector A , $\vec{A} = (1, 0, 3)$

$$\hat{a} = \frac{\vec{A}}{|A|} = \frac{(1,0,3)}{\sqrt{10}} \rightarrow \hat{a} = 0.32\,\hat{a}_x + 0.95\,\hat{a}_z$$

Problem 3.3

Let $\vec{A} = \vec{P}_3 - \vec{P}_2 = (2\hat{a}_x - 2\hat{a}_y - 4\hat{a}_z) - (4\hat{a}_x - 4\hat{a}_y + 4\hat{a}_z)$
$= -2\hat{a}_x + 6\hat{a}_y - 8\hat{a}_z$

and

$\vec{B} = \vec{P}_1 - \vec{P}_2 = (0\,\hat{a}_x + 4\hat{a}_y + 4\hat{a}_z) - (4\hat{a}_x - 4\hat{a}_y + 4\hat{a}_z)$
$= -4\hat{a}_x + 8\hat{a}_z$

parallelogram with sides $\vec{A}$, $\vec{B}$

$\vec{A} \times \vec{B} = AB \sin\theta_{AB}\, \hat{a}_{AB}$

area of the parallelograms 2 times the area of the triangle,

which is 2 * height * base = $2 * A \sin\theta_{AB} * B$

$\therefore$ triangle area $= \frac{1}{2} |\vec{A} \times \vec{B}|$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -2 & 6 & -8 \\ -4 & 0 & 8 \end{vmatrix} = 24(2\hat{a}_x + 2\hat{a}_y + \hat{a}_z)$$

$|\vec{A} \times \vec{B}| = 24\sqrt{4+4+1} = 72$

$\rightarrow$ triangle area $= \frac{1}{2} |A \times B| = \underline{36}$

Problem 3.5

a) $A = |\vec{A}| = \sqrt{1+4+9} = \sqrt{14}$, $\hat{a} = \hat{a}_A = \frac{\vec{A}}{A} = \frac{\sqrt{14}}{14}(\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z)$

b) $\vec{B} \cdot \hat{a}_C = \vec{B} \cdot \frac{\vec{C}}{C} = (2\hat{a}_x - 4\hat{a}_y) \cdot \frac{2\hat{a}_y - 4\hat{a}_z}{\sqrt{2^2+4^2}}$

$= \frac{-8}{\sqrt{20}} = \frac{-4}{5}\sqrt{5} = -1.789$

c) $\theta_{AC} = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{C}}{AC}\right) = \cos^{-1}\left(\frac{4+12}{\sqrt{14}\sqrt{20}}\right) = \cos^{-1}\left(\frac{16}{\sqrt{14}\sqrt{20}}\right) = 17.02^\circ$

d) $\vec{A} \times \vec{C} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & -3 \\ 0 & 2 & -4 \end{vmatrix} = (-8\hat{a}_x + 0\hat{a}_y + 2\hat{a}_z) - (-6\hat{a}_x - 4\hat{a}_y + 0\hat{a}_z)$

$= -2\hat{a}_x - 4\hat{a}_y + 2\hat{a}_z$

e) $\vec{A} \cdot (\vec{B} \times \vec{C}) = (\hat{a}_x - 2\hat{a}_y - 3\hat{a}_z) \cdot \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 2 & -4 & 0 \\ 0 & 2 & -4 \end{vmatrix}$

$= (\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z) \cdot 4(4\hat{a}_x + 2\hat{a}_y + \hat{a}_z) = 20$

f) $\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & 3 \\ 16 & 8 & 4 \end{vmatrix} = (8\hat{a}_x - 48\hat{a}_y + 8\hat{a}_z) -$

$(-24\hat{a}_x + 4\hat{a}_y + 32\hat{a}_z) =$

$32\hat{a}_x - 52\hat{a}_y - 24\hat{a}_z$

g) $\hat{a}_x \times \vec{B} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 0 & 0 \\ 2 & -4 & 0 \end{vmatrix} = -4\hat{a}_z$

h) $(\vec{A} \times \hat{a}_y) \cdot \hat{a}_z = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} \cdot \hat{a}_z = (\hat{a}_z + 3\hat{a}_y) \cdot \hat{a}_z = 1$

Problem 3.8

a) $\vec{A}\cdot(\vec{B}\times\vec{C}) = (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z\end{vmatrix}$

$= (A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)\cdot\left[(B_yC_z - B_zC_y)\hat{a}_x + (B_zC_x - B_xC_z)\hat{a}_y + (B_xC_y - B_yC_x)\hat{a}_z\right]$

$= A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x) \quad (1)$

and $\vec{B}\cdot(\vec{C}\times\vec{A}) = B_x(C_yA_z - C_zA_y) + B_y(C_zA_x - C_xA_z) + B_z(C_xA_y - C_yA_x) \quad (2)$

and $\vec{C}\cdot(\vec{A}\times\vec{B}) = C_x(A_yB_z - A_zB_y) + C_y(A_zB_x - A_xB_z) + C_z(A_xB_y - A_yB_x) \quad (3)$

(1) and (2) and (3) are same six product terms, but in different order

b) $\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z \\ A_x & A_y & A_z \\ (B_yC_z - B_zC_y) & (B_zC_x - B_xC_z) & (B_xC_y - B_yC_x)\end{vmatrix}$

$= [A_y(B_xC_y - B_yC_x) - A_z(B_zC_x - B_xC_z)]\hat{a}_x +$
$[A_z(B_yC_z - B_zC_y) - A_x(B_xC_y - B_yC_x)]\hat{a}_y +$
$[A_x(B_zC_x - B_xC_z) - A_y(B_yC_z - B_zC_y)]\hat{a}_z \quad (1)$

$\vec{B}(\vec{A}\cdot\vec{C}) = (B_x\hat{a}_x + B_y\hat{a}_y + B_z\hat{a}_z)(A_xC_x + A_yC_y + A_zC_z) = (A_xB_xC_x + A_yB_xC_y + A_zB_xC_z)\hat{a}_x + (A_xB_yC_x + A_yB_yC_y + A_zB_yC_z)\hat{a}_y + (A_xB_zC_x + A_yB_yC_y + A_zB_zC_z)\hat{a}_z \quad (2)$

$\vec{C}(\vec{A}\cdot\vec{B}) = (A_xB_xC_x + A_yB_yC_x + A_zB_zC_x)\hat{a}_x + (A_xB_xC_y + A_yB_yC_z + A_yB_yC_y + A_zB_zC_y)\hat{a}_y + (A_xB_xC_z + A_yB_yC_z + A_zB_zC_z)\hat{a}_z \quad (3)$

by subtracting (2) from (3), is the same as (1)